Enrollment No: _____ Exam Seat No:_____

C. U. SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Advanced Mathematics

Subject Code: 2TE02AMT3 **Branch: Diploma (All)**

Semester: 2 **Date: 20/09/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) What is the formula to find Area for curve $y = f(x)$ from a to b ?
વક્ર $y = f(x)$ માટે a થી b પરનું ક્ષેત્રફળ શોધવાનું સુત્ર લખો.

$(a)\ \int_a^b y\,dx \quad (b)\ -\int_a^b y\,dx \quad (c)\ \pi\int_a^b y^2\,dx \quad (d)\ None\ of\ these$

b) $\int_{-\pi/2}^{\pi/2} \cos x\ dx =$ _____.

(a) 2 (b) 1 (c) 0 (d) -1

c) If $y = (3x+2)^3$ then $\frac{dy}{dx}$ *is* ...
જો $y = (3x+2)^3$ તો $\frac{dy}{dx}$ ...

(a) $6(3x+1)$ (b) $2(3x+2)$ (c) $6(3x+2)$ (d) $9(3x+2)^2$

d) If $\vec{x} = (2,1,3), \vec{y} = (3,0,3)$ then find $\vec{x}\cdot\vec{y}$
જો $\vec{x} = (2,1,3), \vec{y} = (3,0,3)$ તો $\vec{x}\cdot\vec{y}$ મેળવો.

(a) 16 (b) 15 (c) 14 (d) 13

e) If a distance of a moving particle is given by $s = t^3 - 3t$, Find the velocity at $t = 3\ sec$.
જો પદાર્થનું સ્થાનાંતર $s = t^3 - 3t$ દ્વારા થાય તો $t = 3\ sec$ માટે વેગ શોધો.

(a) 24 (b) 25 (c) 26 (d) 27

f) $\lim_{x\to 0} \frac{\tan 4x}{4x} =$ _____.

(a) 3 (b) $\frac{1}{3}$ (c) 1 (d) 0

g) $\lim_{x\to 1} \frac{x^3-1}{x-1} =$ _____.

(a) 3 (b) 4 (c) 5 (d) 6

h) What is the derivative of $\cos x$? / $\cos x$ નું વિકલન શું થાય?

(a) $\sin x$ (b) $\cos x$ (c) $-\sin x$ (d) 0

i) $\int 5dx =$ ____ $+ c.$

(a) $5\,x$ (b) $6x^2$ (c) 0 (d) 10

j) $\int_0^1 x^3\, dx =$ _____.

$(a)\ \frac{1}{2}\quad (b)\ \frac{1}{3}\quad (c)\ \frac{1}{4}\quad (d)\ \frac{1}{5}$

k) Which of the following is equal to $\int_a^b f(x)dx$?

નીચેનામાંથી $\int_a^b f(x)dx$ ના બરાબર શું થાય ?

$(a)\ \int_a^b f(t)dt\quad (b)\ -\int_b^a f(x)\,dx\quad (c)\ both\ a\ and\ b\quad (d)\ None\ of\ these$

l) $\frac{d(2x)}{dx} =$ _____.

a) 3 (b) 2 (c) 1 (d) 0

m) Find modulus of $v = (3,2,1) + (1,2,3)$

$v = (3,2,1) + (1,2,3)$ નો માનાંક શોધો.

(a) $4\sqrt{3}$ (b) $3\sqrt{4}$ (c) $\sqrt{43}$ (d) $\sqrt{34}$

n) $\int \text{cosec}^2\, x\ dx =$ ____ $+ c.$

(a) $-\cos x$ (b) $\sin x$ (c) $\tan x$ (d) $-\cot x$

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a). Evaluate/ કિંમત શોધો: $\lim_{x\to -1}\frac{2x^3+5x^2+4x+1}{3x^3+5x^2+x-1}$. **(05)**

b). Evaluate/ કિંમત શોધો: $\lim_{x\to 3}\frac{\sqrt{x+5}-2\sqrt{2}}{\sqrt{x-1}-\sqrt{2}}$. **(05)**

c). Evaluate/ કિંમત શોધો: 1) $\lim_{x\to 0}\frac{\sin 5x}{\sin 10x}$. 2) $\lim_{x\to \frac{\pi}{2}}(sec\,x - tan\,x)$. **(04)**

Q-3 **Attempt all questions** **(14)**

a). If $\vec{a}=2\hat{i}-3\hat{j}+3\hat{k}, \vec{b}=\hat{i}-3\hat{j}-2\hat{k}$, and $\vec{c}=3\hat{i}+2\hat{j}+2\hat{k}$, then find modulus of $2\vec{a}+3\vec{b}-\vec{c}$. **(05)**

જો $\vec{a}=2\hat{i}-3\hat{j}+3\hat{k}, \vec{b}=\hat{i}-3\hat{j}-2\hat{k}$, અને $\vec{c}=3\hat{i}+2\hat{j}+2\hat{k}$, તો $2\vec{a}+3\vec{b}-\vec{c}$ નો માનાંક શોધો.

b). Find unit vector which is perpendicular to the vectors $\vec{x}=(3,1,2)$ and $\vec{y}=(2,1,1)$. **(05)**

$\vec{x}=(3,1,2)$ અને $\vec{y}=(2,1,1)$ ને લંબ હોય તેવો એકમ શદિશ મેળવો.

c). Find box product of the vectors $-8\hat{i}-\hat{j}+3\hat{k}$, $-\hat{i}+4\hat{j}+3\hat{k}$, and $\vec{c}=-4\hat{i}-6\hat{j}-2\hat{k}$. Also find $\vec{a}\cdot\vec{b}$. **(04)**

બોક્ષ પ્રોડક્ટ મેળવો, જ્યાં શદિશ $\vec{a}=-8\hat{i}-\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+4\hat{j}+3\hat{k}$, અને $\vec{c}=-4\hat{i}-6\hat{j}-2\hat{k}$. તેમજ $\vec{a}\cdot\vec{b}$ મેળવો.

Q-4 **Attempt all questions** **(14)**

a). Find the derivative of $f(x)=x^n$ using definition. **(05)**

વ્યાખ્યાની મદદથી $f(x)=x^n$ નુ વિકલન મેળવો.

b). Find $\frac{dy}{dx}$ / $\frac{dy}{dx}$ મેળવો: $y=\frac{a+b\,sin\,x}{a\,sin\,x+b}$. **(05)**

c). Find $\frac{dy}{dx}$ / $\frac{dy}{dx}$ મેળવો: $y=\frac{e^{ax}}{a^2+b^2}(a\,cos\,bx+b\,sin\,bx)$. **(04)**

Q-5 **Attempt all questions** **(14)**

a). If $x=e^{\theta}\,sin\,\theta$ and $y=e^{\theta}\,cos\,\theta$, show that $\frac{dy}{dx}=\frac{1-tan\,\theta}{1+tan\,\theta}$. **(05)**

જો $x=e^{\theta}\,sin\,\theta$ અને $y=e^{\theta}\,cos\,\theta$, તો બતાવો કે $\frac{dy}{dx}=\frac{1-tan\,\theta}{1+tan\,\theta}$.

b). $y=e^{ax}\sin bx$, prove that $\frac{d^2y}{dx^2}-2a\frac{dy}{dx}+(a^2+b^2)y=0$. **(05)**

$y=e^{ax}\sin bx$, સાબિત કરો કે $\frac{d^2y}{dx^2}-2a\frac{dy}{dx}+(a^2+b^2)y=0$.

c). Find the derivative/ વિકલન મેળવો **(04)**

$i).\ y = x^3 + 4x^2 - 2x + 1 \quad ii).\ y = e^x + x^e + e^e$

Q-6 **Attempt all questions** **(14)**

a). The distance of a moving particle $s = t^3 + 3t, t > 0$, **(06)**

(i) Find the velocities and acceleration at $t = 3$ sec.

(ii) When velocity and acceleration become equal?

પદાર્થનું સ્થાનાંતર $s = t^3 + 3t$ છે તો

(i) $t = 3$ sec માટે વેગ અને પ્રવેગ મેળવો.

(ii) વેગ અને પ્રવેગ ક્યારે સમાન બનશે.

b). Integrate: $\int \left[\sqrt{(1 + sin2x)} + \sqrt{\frac{1 + \cos 2x}{1 - \cos 2x}}\right] dx$ **(05)**

c). Evaluate/ કિંમત શોધો: $\lim_{x \to 3} \frac{x^{10} - 10^{10}}{x - 10}$ **(03)**

Q-7 **Attempt all questions** **(14)**

a). Find the volume generated by the circle $x^2 + y^2 = 16$, revolving about the *x*-axis. **(07)**

X- અક્ષની આસપાસ પરિભ્રમણ કરતા વર્તુળ $x^2 + y^2 = 16$ દ્વારા રચાતું કદ શોધો.

b). Show that/ બતાવો કે: $\int_0^{\pi/2} \frac{\sqrt[3]{sinx}}{\sqrt[3]{sinx} + \sqrt[3]{\cos x}} dx = \frac{\pi}{4}$. **(04)**

c). Evaluate/ કિંમત શોધો: $\int_1^2 \frac{\mathrm{x}}{1 + \mathrm{x}^2} dx$ **(03)**

Q-8 **Attempt all questions** **(14)**

a). If $\vec{a} = 2\hat{\imath} - 3\hat{\jmath},\ \vec{b} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$, then obtain $\left|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})\right|$. **(05)**

જો $\vec{a} = 2\hat{\imath} - 3\hat{\jmath},\ \vec{b} = 4\hat{\imath} + 3\hat{\jmath} - \hat{k}$, તો $\left|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})\right|$ ની કિંમત શોધો.

b). Evaluate/ કિંમત શોધો: $\lim_{x \to \infty} \frac{x^3 + x - 3}{2x^3 + 3x^2 + 2x + 1}$. **(05)**

c). Evaluate/ કિંમત શોધો: $\int_{-2}^{2} \mathrm{x}^5 (1 - \mathrm{x})^{\frac{3}{2}}\ \mathrm{dx}$ **(04)**